



St Peter's School

Written Calculations Policy

Updated February 2018

Rationale

This policy outlines a model progression through written strategies for addition, subtraction, multiplication and division in line with the new National Curriculum commencing September 2014. Through the policy, we aim to link key manipulatives and representations in order that the children can be vertically accelerated through each strand of calculation. A school wide policy, such as this, can ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum 2014 and in line with school policy.

The importance of mental mathematics

While this policy focuses on written calculations in mathematics, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.

To add and subtract successfully, children should be able to:

- recall all addition pairs to $9 + 9$ and number bonds to 10
- recognise addition and subtraction as inverse operations
- add mentally a series of one digit numbers (e.g. $5 + 8 + 4$)
- add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. $600 + 700$, $160 - 70$)
- partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into $70 + 4$ or $60 + 14$)
- use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:

- add and subtract accurately and efficiently
- recall multiplication facts to $12 \times 12 = 144$ and division facts to $144 \div 12 = 12$
- use multiplication and division facts to estimate how many times one number divides into another etc.
- know the outcome of multiplying by 0 and by 1 and of dividing by 1
- understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
- recognise factor pairs of numbers (e.g. that $15 = 3 \times 5$, or that $40 = 10 \times 4$) and increasingly able to recognise common factors
- derive other results from multiplication and division facts and multiplication and division by 10 or 100 (and later 1000)
- notice and recall with increasing fluency inverse facts
- partition numbers into 100s, 10s and 1s or multiple groupings
- understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
- understand the effects of scaling by whole numbers and decimal numbers or fractions
- understand correspondence where n objects are related to m objects
- investigate and learn rules for divisibility

Progression in addition and subtraction

Addition and subtraction are connected.

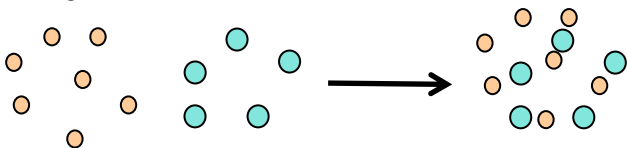
Part	Part
Whole	

Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

Addition

Combining two sets (aggregation)

Putting together – two or more amounts or numbers are put together to make a total
 $7 + 5 = 12$



Count one set, then the other set. Combine the sets and count again. Starting at 1.
 Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1.

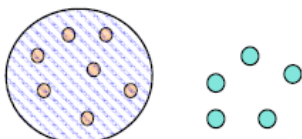


Combining two sets (augmentation)

This stage is essential in starting children to calculate rather than counting

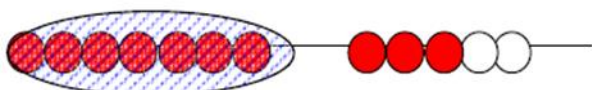
Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.

Counters:



Start with 7, then count on 8, 9, 10, 11, 12

Bead strings:



Make a set of 7 and a set of 5. Then count on from 7.

Multilink Towers:



Number tracks:



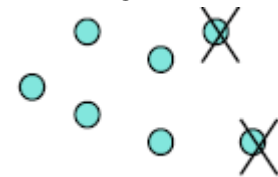
Start on 5 then count on 3 more

Subtraction

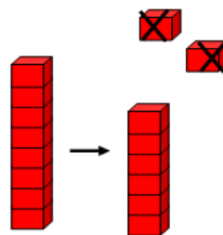
Taking away (separation model)

Where one quantity is taken away from another to calculate what is left.

$$7 - 2 = 5$$



Multilink towers - to physically take away objects.



Finding the difference (comparison model)

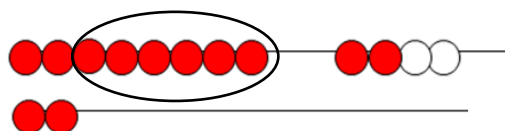
Two quantities are compared to find the difference.

$$8 - 2 = 6$$

Counters:

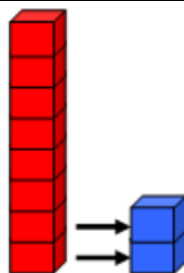


Bead strings:



Make a set of 8 and a set of 2. Then count the gap.

Multilink Towers:



Number tracks:



Start with the smaller number and count the gap to the larger number.

1 set within another (part-whole model)

The quantity in the whole set and one part are known, and may be used to find out how many are in the unknown part.

$$8 - 2 = 6$$

Counters:



Bead strings:

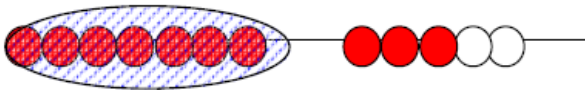
$$8 - 2 = 6$$



Bridging through 10s

This stage encourages children to become more efficient and begin to employ known facts.

Bead string:



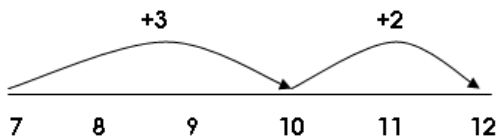
$7 + 5$ is decomposed / partitioned into $7 + 3 + 2$. The bead string illustrates 'how many more to the next multiple of 10?' (children should identify how their number bonds are being applied) and then 'if we have used 3 of the 5 to get to 10, how many more do we need to add on? (ability to decompose/partition all numbers applied)

Number track:

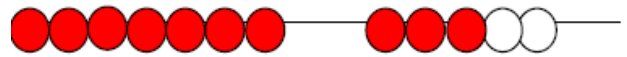


Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number line

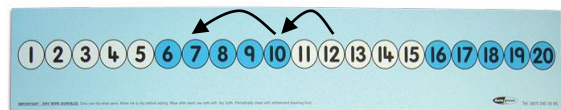


Bead string:



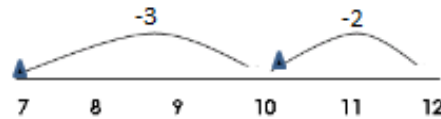
$12 - 7$ is decomposed / partitioned in $12 - 2 - 5$. The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back? (ability to decompose/partition all numbers applied)

Number Track:



Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number Line:



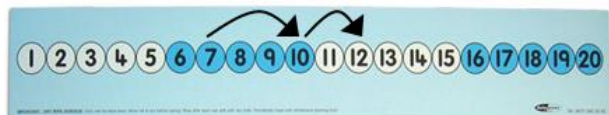
Counting up or 'Shop keepers' method

Bead string:

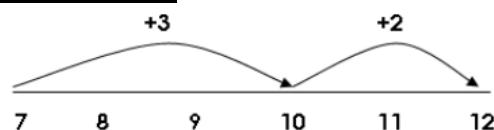


$12 - 7$ becomes $7 + 3 + 2$. Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'.

Number Track:



Number Line:

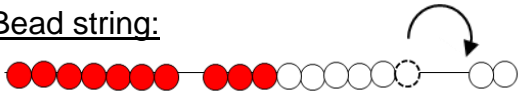


Compensation model (adding 9 and 11)

This model of calculation encourages efficiency and application of known facts (how to add ten)

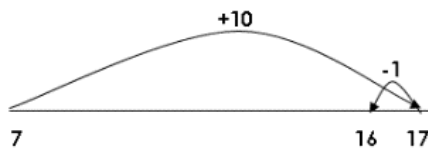
$7 + 9$

Bead string:



Children find 7, then add on 10 and then adjust by removing 1.

Number line:



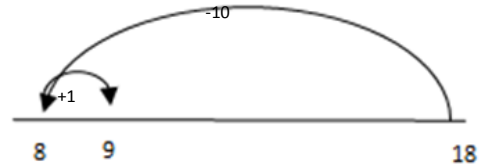
$18 - 9$

Bead string:



Children find 18, then subtract 10 and then adjust by adding 1.

Number line:



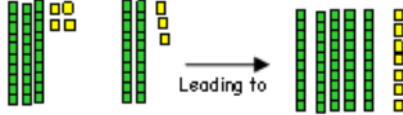
Working with larger numbers
Tens and ones + tens and ones

Ensure that the children have been transitioned onto Dienes apparatus and understand the abstract nature of the single 'tens' sticks and 'hundreds' blocks

Partitioning (Aggregation model)

$$34 + 23 = 57$$

Dienes equipment:

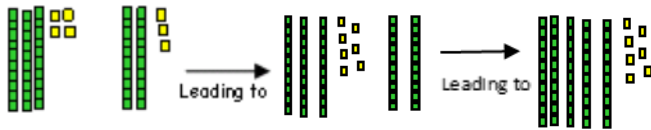


Children create the two sets with Dienes equipment and then combine; ones with ones, tens with tens.

Partitioning (Augmentation model)

Dienes equipment:

Encourage the children to begin counting from the first set of ones and tens, avoiding counting from 1. Beginning with the ones in preparation for formal column method.



Number line:



Use a blank number line to add using partitioning. Each jump to represent each part of the larger number.

Take away (Separation model)

$$57 - 23 = 34$$

Dienes equipment:

Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.



Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the ones are greater than 10? Introduce the term '**exchange**'. Using the Dienes equipment, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Expanded method

Starting with:

$$35 + 24 = 59$$

$$\begin{array}{r} 30 + 5 \\ +20 + 4 \\ \hline 50 + 9 = 59 \end{array}$$

Leading to:

$$\begin{array}{r} 37 \\ + 15 \\ \hline 12 \text{ (7+5)} \\ +40 \text{ (30 + 10)} \\ \hline 52 \end{array}$$

Starting by adding the ones together and then the tens. These answers are then combined together.

Expanded method

Starting with:

$$34 - 21 =$$

$$\begin{array}{r} 30 - 4 \\ -20 - 1 \\ \hline 10 + 3 = 13 \end{array}$$

Leading to:

$$52 - 31 =$$

Each number is partitioned and then ones and tens are then subtracted.

$$\begin{array}{r} 50 \quad 2 \\ - 30 \quad 1 \\ \hline 20 \quad 1 \end{array}$$

The answer can then be combined together.

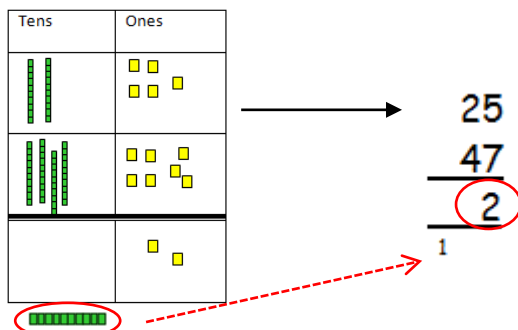
$$\text{So } 52 - 31 = 21$$

$$224 - 139 =$$

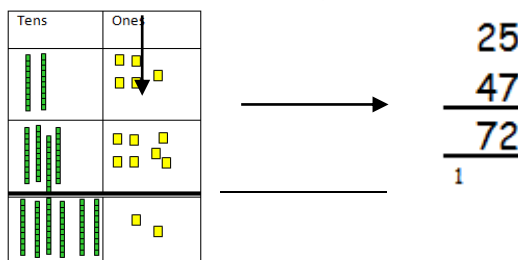
$$\begin{array}{r} 100 \quad 200 \quad 110 \quad 20 \quad 14 \\ - \quad 100 \quad 30 \quad 9 \\ \hline 100 + 80 + 5 = 185 \end{array}$$

Compact method

We would expect most children using this by the end of Year 4.

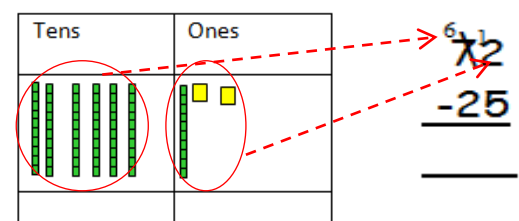
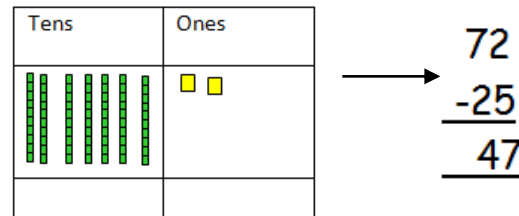


Leading to



Compact decomposition

We would expect most children using this by the end of Year 3.



$$\begin{array}{r} 367 \\ + 85 \\ \hline 452 \\ 11 \end{array}$$

Tens	Ones

$$\begin{array}{r} 612 \\ - 25 \\ \hline 47 \end{array}$$

This is then developed into multiplying and dividing decimals and money.

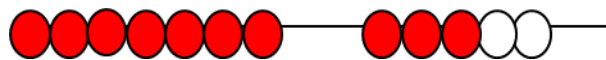
Vertical acceleration

By returning to earlier manipulative experiences children are supported to make links across mathematics, encouraging 'If I know this...then I also know...' thinking.

Decimals

Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:



Each bead represents 0.1, each different block of colour equal to 1.0

Dienes equipment:



Addition of decimals

Aggregation model of addition

Counting both sets – starting at zero.

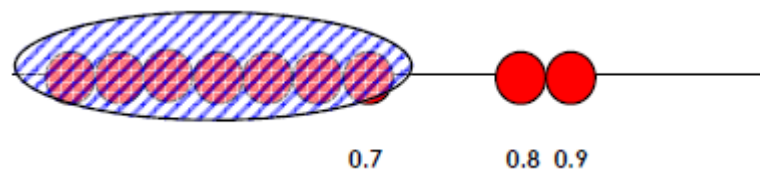
$$0.7 + 0.2 = 0.9$$



Augmentation model of addition

Starting from the first set total, count on to the end of the second set.

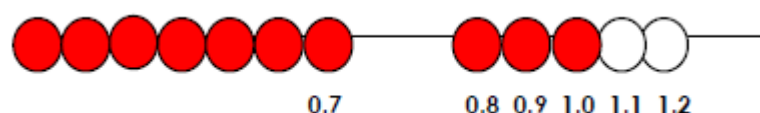
$$0.7 + 0.2 = 0.9$$



Bridging through 1.0

Encouraging connections with number bonds.

$$0.7 + 0.5 = 1.2$$



Partitioning

Subtraction of decimals

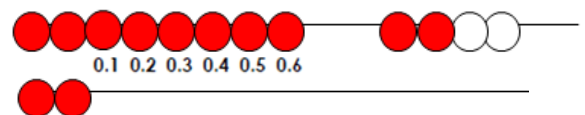
Take away model

$$0.9 - 0.2 = 0.7$$



Finding the difference (or comparison model):

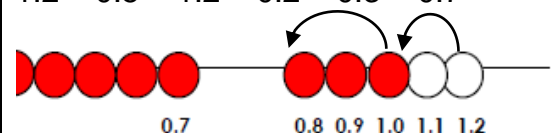
$$0.8 - 0.2 =$$



Bridging through 1.0

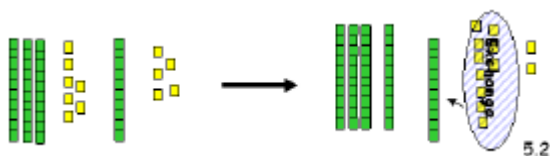
Encourage efficient partitioning.

$$1.2 - 0.5 = 1.2 - 0.2 - 0.3 = 0.7$$

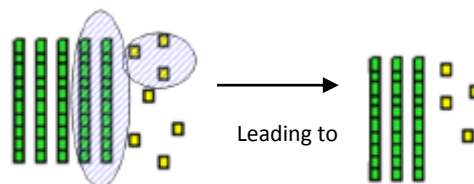


Partitioning

$$3.7 + 1.5 = 5.2$$



$$5.7 - 2.3 = 3.4$$



Progression within addition:

1. No exchange
2. Extra digit in the answer
3. Exchanging ones to tens
4. Exchanging tens to hundreds
5. Exchanging ones to tens and tens to hundreds
6. More than two numbers in calculation
7. As 6 but with different number of digits
8. Decimals up to 2 decimal places (same number of decimal places)
9. Add two or more decimals with a range of decimal places

Progression within subtraction:

1. No exchange
2. Fewer digits in the answer
3. Exchanging tens for ones
4. Exchanging hundreds for tens
5. Exchanging hundreds to tens and tens to ones
6. As 5 but with different number of digits
7. Decimals up to 2 decimal places (same number of decimal places)
8. Subtract two or more decimals with a range of decimal places

Progression in Multiplication and Division

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			



The following array, consisting of four columns and three rows, could be used to represent the number sentences: -

$$3 \times 4 = 12,$$

$$4 \times 3 = 12,$$

$$3 + 3 + 3 + 3 = 12,$$

$$4 + 4 + 4 = 12.$$



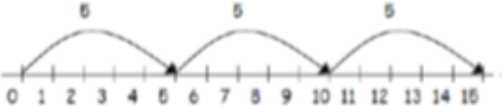



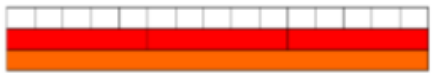

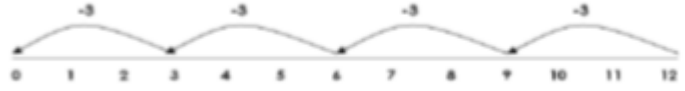
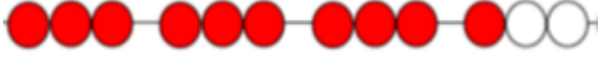
And it is also a model for division

$$12 \div 4 = 3$$

$$12 \div 3 = 4$$

$$12 - 4 - 4 - 4 = 0$$

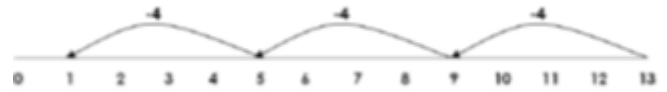
$$12 - 3 - 3 - 3 - 3 = 0$$

Multiplication	Division
<p>Early experiences</p> <p>Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.</p> 	<p>Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.</p> 
<p>Repeated addition (repeated aggregation)</p> <p>3 times 5 is $5 + 5 + 5 = 15$ or 5 lots of 3 or 5×3</p> <p>Children learn that repeated addition can be shown on a number line.</p>  <p>Children learn that repeated addition can be shown on a bead string.</p> 	<p>Sharing equally</p> <p>6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.</p>  <p>Grouping or repeated subtraction</p> <p>There are 6 sweets. How many people can have 2 sweets each?</p> 
<p>Scaling</p> <p>This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles. This can be written as:</p> <p>$1 + 1 + 1 = 3$ □ scaled up by 5 □ $5 + 5 + 5 = 15$</p>  <p>For example, find a ribbon that is 4 times as long as the blue ribbon.</p>  <p>We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to $3 \times 0.5 = 1.5$.</p>	<p>Repeated subtraction using a bead string or number line</p> <p>$12 \div 3 = 4$</p>   <p>The bead string helps children with interpreting division calculations, recognising that $12 \div 3$ can be seen as 'how many 3s make 12?'</p>

Grouping involving remainders

Children move onto calculations involving remainders.

$$13 \div 4 = 3 \text{ r}1$$



Or using a bead string see above.

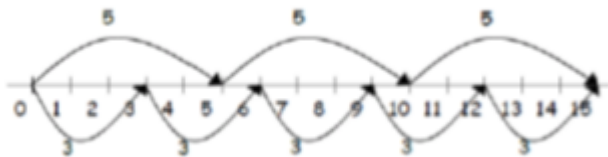
Commutativity

Children learn that 3×5 has the same total as 5×3 .

This can also be shown on the number line.

$$3 \times 5 = 15$$

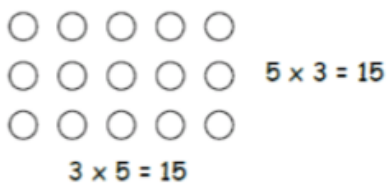
$$5 \times 3 = 15$$



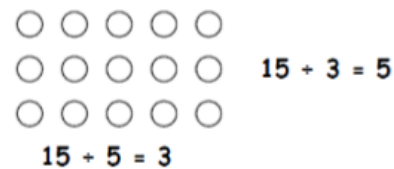
Children learn that division is **not** commutative and link this to subtraction.

Arrays

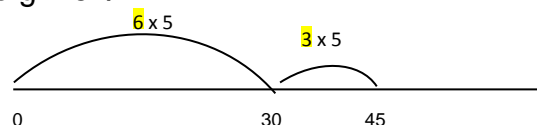
Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number.



Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities.

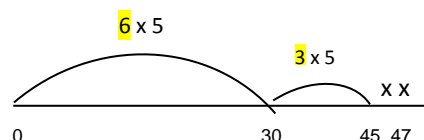


Division on a blank number line
e.g. $45 \div 5 =$



Therefore $6 + 3 = 9$ so $45 \div 5 = 9$

$$47 \div 5 =$$



Therefore $6 + 3 = 9 \text{ r}2$

$$\text{So } 47 \div 5 = 9 \text{ r}2$$

Inverse operations

Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

$$3 \times 4 = 12$$

$$4 \times 3 = 12$$

$$12 \div 3 = 4$$

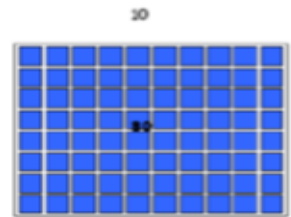
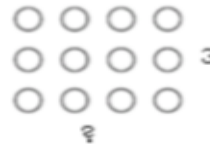
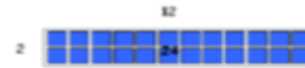
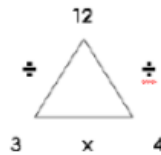
$$12 \div 4 = 3$$

Children use symbols to represent unknown

numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

$$\square \times 5 = 20 \quad 3 \times \Delta = 18 \quad O \times \square = 32$$

$$24 \div 2 = \square \quad 15 \div O = 3 \quad \Delta \div 10 = 8$$



This can also be supported using arrays: e.g. $3 \times ? = 12$

To be successful in calculation learners must have plenty of experiences of being flexible with partitioning, as this is the basis of distributive and associative law.

Associative law

E.g. $3 \times (3 \times 4) = 36$



(multiplication only) :-

The principle that if there are three numbers to multiply these can be multiplied in any order.

Distributive law (multiplication):-

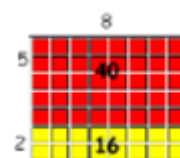
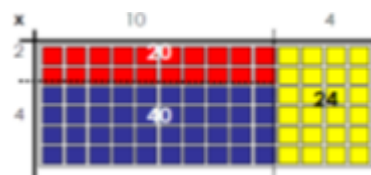
E.g. $6 \times 14 = (2 \times 10) + (4 \times 10) + (4 \times 6) = 20 + 40 + 24 = 84$

This law allows you to distribute a multiplication across an addition or subtraction.

Distributive law (division):-

E.g. $56 \div 8 = (40 \div 8) + (16 \div 8) = 5 + 2 = 7$

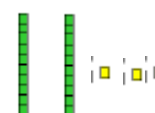
This law allows you to distribute a division across an addition or subtraction.



Arrays leading into the grid method

Children continue to use arrays and partitioning, where appropriate, to prepare them for the grid method of multiplication. Arrays can be represented as 'grids' in a shorthand version and by using dienes apparatus show multiples of ten, hundred etc.

24×3



Grid method

We would expect most children to be using this by the end of Year 3

This written strategy is introduced for the multiplication of TO x O to begin with. It may require column addition methods to calculate the total.

x	300	40	6	
3	900	120	18	= 1038

The vertical method- 'chunking' leading to long division

We would expect most children to be using this by the end of Year 3

This could be modelled using dienes apparatus. Starting with 78 and then by chunking.

$$78 \div 3 =$$

$$\begin{array}{r} 78 \\ - 30 \\ \hline 48 \\ - 30 \\ \hline 18 \\ - 18 \\ \hline 0 \end{array} \quad \begin{array}{l} (10 \times 3) \\ (10 \times 3) \\ (6 \times 3) \end{array}$$

$$\text{So } 78 \div 3 = 10 + 10 + 6 = 26$$

Expanded Method

This written strategy is the bridge between Grid method and the short method.

$$\begin{array}{r} 353 \\ \times 4 \\ \hline 12 \text{ (3 x 4)} \\ 200 \text{ (50 x 4)} \\ 1200 \text{ (300 x 4)} \\ \hline 1412 \end{array} \quad \begin{array}{l} \text{As the children become} \\ \text{more confident they} \\ \text{may omit writing each} \\ \text{step of the calculation} \end{array}$$

Short multiplication — multiplying by a single digit

$$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \end{array} \quad \begin{array}{r} 24 \\ \times 6 \\ \hline 144 \end{array}$$

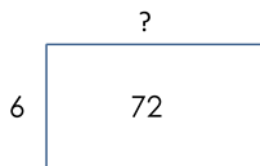
Short division — dividing by a single digit

$$\begin{array}{r} 3 \\ 4 \overline{) 136} \\ \underline{12} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

<p><u>Progression within short multiplication</u></p> <ol style="list-style-type: none"> 1. TO x O no exchange 2. TO x O extra digit in the answer 3. TO x O with exchange of ones into tens 4. HTO x O no exchange 5. HTO x O with exchange of ones into tens 6. HTO x O with exchange of tens into hundreds 7. HTO x O with exchange of ones into tens and tens into hundreds 8. As 4-7 but with greater number digits x O 9. O.t x O no exchange 10. O.t with exchange of tenths to ones 11. As 9 - 10 but with greater number of digits which may include a range of decimal places x O 	<p><u>Progression within short division</u></p> <ol style="list-style-type: none"> 1. TO ÷ O no exchange no remainder 2. TO ÷ O no exchange with remainder 3. TO ÷ O with exchange no remainder 4. TO ÷ O with exchange, with remainder 5. Zero in the quotient e.g. $816 \div 4 = \mathbf{204}$ 6. As 1-5 HTO ÷ O 7. As 1-5 greater number of digits ÷ O 8. As 1-5 with a decimal dividend e.g. $7.5 \div 5$ or $0.12 \div 3$ 9. Where the divisor is a two digit number <p>See below for gradation of difficulty with remainders</p>
	<p><u>Dealing with remainders</u></p> <p>Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly.</p> <p>e.g.:</p> <ul style="list-style-type: none"> · I have 62p. How many 8p sweets can I buy? · Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed? <p><u>Gradation of difficulty for expressing remainders</u></p> <ol style="list-style-type: none"> 1. Whole number remainder 2. Remainder expressed as a fraction of the divisor 3. Remainder expressed as a simplified fraction 4. Remainder expressed as a decimal
<p><u>Long multiplication—multiplying by more than one digit</u></p> <p>Children will refer back to grid method by using place value counters or Dienes equipment with no exchange and using synchronised modelling of written recording as a long multiplication model before moving to TO x TO etc.</p>	<p><u>Long division —dividing by more than one digit</u></p> <p>Children should be reminded about partitioning numbers into multiples of 10, 100 etc. before recording as either:-</p> <ol style="list-style-type: none"> 1. Chunking model of long division using Dienes equipment 2. Sharing model of long division using place value counters <p>See the following pages for exemplifications</p>

Chunking model of long division using Dienes equipment

This model links strongly to the array representation; so for the calculation $72 \div 6 = ?$ - one side of the array is unknown and by arranging the Dienes equipment to make the array we can discover this unknown. The written method should be written alongside the equipment so that children make links.

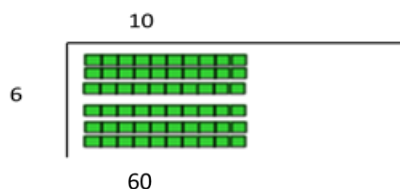


Begin with divisors that are between 5 and 9

$$72 \div 6 = 12$$



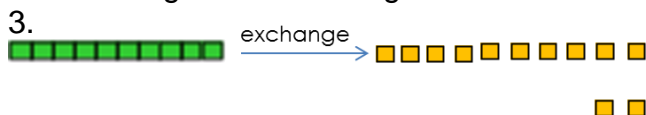
1. Make a rectangle where one side is 6 (the number dividing by) – grouping 6 tens



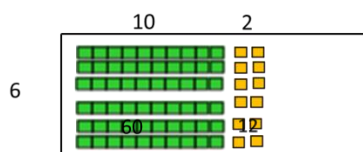
After grouping 6 lots of 10 (60) we have 12 left over



2. Exchange the remaining ten for ten ones



4. Complete the rectangle by grouping the remaining ones into groups of 6



$$6 \overline{) 72}$$

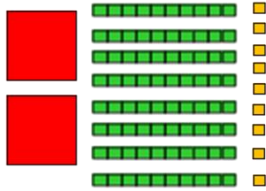
$$\begin{array}{r} 1 \\ 6 \overline{) 72} \\ \underline{- 60} \quad (10 \times) \\ 12 \end{array}$$

$$\begin{array}{r} 12 \\ 6 \overline{) 72} \\ \underline{- 60} \quad (10 \times) \\ 12 \\ \underline{- 12} \quad (2 \times) \\ 0 \end{array}$$

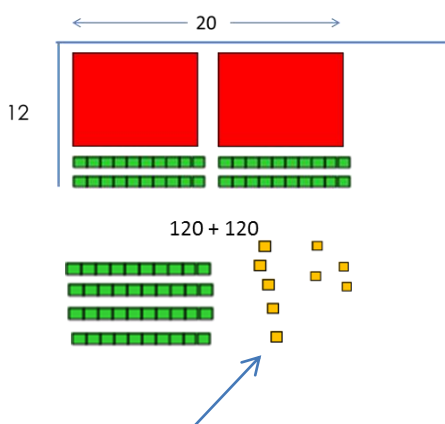
Move onto working with divisors between 11 and 19

Children may benefit from practise to make multiples of tens using the hundreds and tens and tens and ones.

$$289 \div 12$$

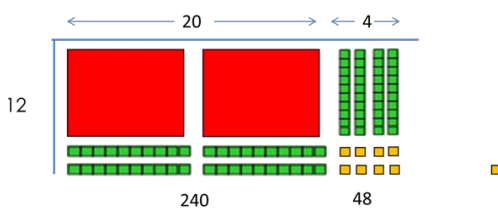


1. Make a rectangle where one side is 12 (the number dividing by) using hundreds and tens



With 49 remaining

2. Make groups of 12 using tens and ones



No more groups of 12 can be made and 1 remains

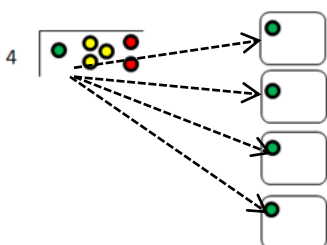
$$12 \overline{) 289}$$

$$\begin{array}{r} 2 \\ 12 \overline{) 289} \\ - \underline{240} \quad (20 \times) \\ 49 \end{array}$$

$$\begin{array}{r} 24 \text{ r}1 \\ 12 \overline{) 289} \\ - \underline{240} \quad (20 \times) \\ 49 \\ - \underline{48} \quad (4 \times) \\ 1 \end{array}$$

Sharing model of long division using dienes apparatus

Starting with the most significant digit, share the hundreds. The writing in brackets is for verbal explanation only. (Dots represent dienes apparatus)



$$\begin{array}{r} 1 \\ 4 \overline{) 532} \\ \underline{4} \quad (4 \text{ hundreds used}) \\ 1 \quad (1 \text{ hundred left}) \end{array}$$

Diagram illustrating the exchange of 4 hundreds for 4 tens. The top part shows a box labeled '1' containing 4 hundreds blocks (green, yellow, yellow, red) and a curved arrow pointing to a box containing 4 tens blocks (all yellow). The bottom part shows a vertical number line with the following values: 532, 4413, and 13. The text '(4 hundreds used)' is written next to the 4413, and '(13 tens)' is written next to the 13.

